STOCHASTIC OPTIMIZATION OF VIOLIN TOPS

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Abstract

The objective of the work presented is to show that it is possible, through changes in variables as thickness distribution and arching, and by optimally placed discrete masses to compensate for the changes in vibration properties caused by a variation of the material parameters in a violin top. The changes in the variables are determined trough the stochastic optimization method simulated annealing (SA) [1]. A code based on this algorithm is linked together with a modified version of the finite element code FEMP [2] to achieve an optimization program, which incorporates structural design changes in an automatic fashion.

INTRODUCTION

It is possible to change the vibration properties, as mode shapes and eigenfrequencies, and the characteristics of the sound emanating from a vibrating structure by changing structural design variables such as geometric dimensions [3], shell thickness [4], material parameters [5], discrete masses [6], and, for fiber reinforced material, fiber orientation [7]. Of course, changes to one or more of these variables will results in changes to other structural characteristics. To find the best design, i.e. the one that satisfies all the demands put upon is a question of optimization. This often requires a multidisciplinary approach i.e. analytical tools from different disciplines must be used in concert. The material parameters, which have a natural distribution, have great influence on the vibration properties of the violin plates. Earlier studies on blanks for violin tops and backs gives some idea of the sensitivity of the vibration properties with respect to the material parameters. In the present work we use optimization to give some suggestions on how to compensate, through changes in thickness distribution, rise of the arch, and, optimally placed discrete masses for variations in the material parameters, that is: is it possible to change the material parameters and retrieve the initial vibration properties. The search for the optimum solution can be performed in many different ways. A common approach is to use mathematical programming techniques. This technique has been used in earlier analysis [7] where, a gradient-based method called MMA [8] was used to achieve optimization. MMA has been used with great success for a variety of problems. Another method, which is conceptually quite different from the mathematical programming techniques, is to optimize using some form of natural selection process. One such technique is the simulated annealing, SA, a stochastic method based on the simulation of metal (or solids) annealing [9]. Annealing is the physical process of heating up a solid and then cooling it down slowly. The slow and controlled cooling of the solid ensures proper solidification with a highly ordered crystalline state. At high temperature the atoms in the heated material have high energies and more freedom to arrange themselves. Annealing results in a material with an atom arrangement that corresponds to the lowest internal energy. There are many other optimization methods, such as genetic algorithms, neural network and so on, that is based on natural selection of solutions to achieve an optimum. In this paper simulated annealing is used as the optimization algorithm. A code based on this algorithm [1] is linked together with a modified version of the finite element code FEMP [2] to achieve an optimization program, which incorporates structural design changes in an automatic fashion. In this paper, the thickness variation, the rise of the arch of the top and optimally sized masses placed at specific locations are used as variables.

PROBLEM DEFINITION

For the purpose of optimization the material parameters for the violin top was replaced and the objective with the analysis was to recover the initial eigenfrequencies by altering the thickness distribution and the rise of the arch throughout the top and by placing optimally sized masses at specific points on the top. The optimization involves modal analysis by use of FE-calculations and the top was discretizised with triangular orthotropic shell elements according to Figure 1. In the FE-analyses the nodes along the bolded line in Figure 1 was simply supported, i.e. prevented to move perpendicular to the plane but free in all other directions. In the optimization the first three eigenmodes according to Figure 2 were studied. The objective function was formulated as:

$$\textit{Minimize:} \qquad z = \alpha_k \cdot \left(f_k^{initial} - f_k^{actual}\right)^2, \qquad k = 1, 3 \tag{1}$$

where $f_k^{initial}$ are the eigenfrequencies for the top with initial material parameters and f_k^{actual} are the eigenfrequencies with new material parameters and with actual variable set. α_k are the so-called penalty parameters. The only constraints used in this investigation are on the variables.

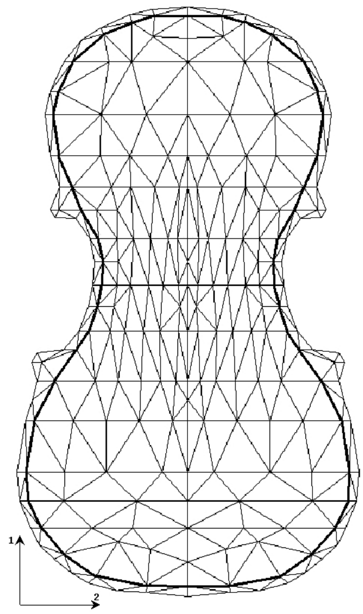

Fig. 1. Discretization of violin top.

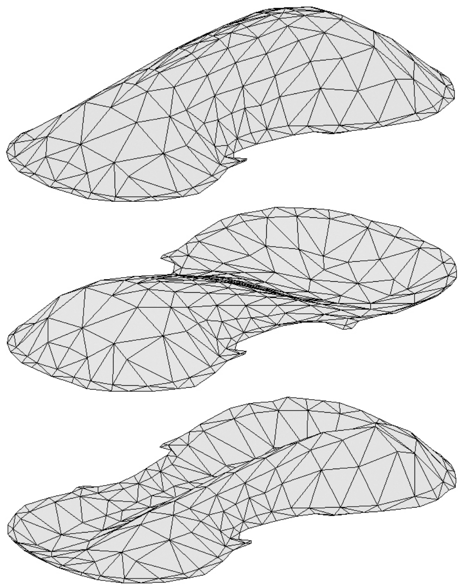

Fig. 2. Studied eigenmodes. Top: mode 1, middle: mode 2, bottom: mode 3.

The formulation according to (1) with constraints only on the variables is well suited for simulated annealing which is a nongradient (zeroth-order) stochastic optimization technique based on random evaluation of the objective function in such a way that transitions away from local minimum are possible. Although the method usually requires a large number of function evaluations to find the optimum design, it will find the global optimum with a high probability even for problems with numerous local minima [9].

RESULTS

With shell thickness and rise of the arch as variables a maximum variation of ±10% from the initial variable value was allowed. The initial material (spruce) parameters are taken as: $E_1 = 9.567 \cdot 10^9$ Pa, $E_2 = 5.789 \cdot 10^8$ Pa, $G_{12} = 7.08 \cdot 10^8$ Pa, $\nu_{12} = 0.03$, and $\rho = 410$ kg/m^3. With initial thickness, initial rise and these material parameters the studied eigenfrequencies were determined to: $f_1 = f_1^{initial} =$ 283.31 Hz, $f_2 = f_2^{initial} =$ 510.79 Hz, and $f_3 = f_3^{initial} =$ 542.54 Hz.

The new material parameters were chosen with respect to a 5% decrease of the density, from 410 to 389.5 kg/m^3. This affected the other parameters and resulted in new material parameters according to: $E_1 = 9.092 \cdot 10^9$ Pa, $E_2 = 4.963 \cdot 10^8$ Pa, $G_{12} = 6.726 \cdot 10^8$ Pa, $\nu_{12} = 0.03$, and as mentioned above ρ = 389.5 kg/m^3. The new material set together with the initial thickness and initial rise resulted in the following eigenfrequencies: f_1 = 279.90 Hz (-1.2%), f_2 = 505.02 Hz (-1.1%), and f_3 = 532.60 Hz (-1.8%). These are the first f_k^{actual} , (k = 1, 3), in the optimization process and with the penalty parameters α_k taken as 1.0 it follows that the objective function is z = 143.72 in the beginning of the optimization process.

Thickness distribution

In this case the shell thickness at nodes along the bolded line in Figure 1 are collected to one variable and the thickness at nodes outside the line collected to another variable. The shell thicknesses at all other nodes are separately variables. Utilization of symmetry with respect to the vertical mid-axis gives the total number of thickness variables to 68. The optimization process converged to the following eigenfrequencies: $f_1 = f_1^{\text{optimal}}$ = 283.30 Hz (-0.004%), $f_2 = f_2^{\text{optimal}}$ = 510.78 Hz (-0.002%), and $f_3 = f_3^{\text{optimal}}$ = 542.53 Hz (-0.002%). The change in thickness distribution is illustrated in Figure 3 where the values are in meter and shall be added to the initial thickness distribution to get the distribution at optimum.

Rise of the arch

Here the rise of the arch for the top was taken as variable. The rise for the nodes along the bolded line in Figure 1 and the nodes outside this line are kept constant during the optimization process. The rise at all other nodes is separately variables. Also here symmetry with respect to the vertical mid-axis is utilized which gives the total number of variables to 66. The optimization process converged to the following eigenfrequencies: : $f_1 = f_1^{\text{optimal}}$ = 283.38 Hz (+0.025%), $f_2 = f_2^{\text{optimal}}$ = 511.04 Hz (+0.049%), and $f_3 = f_3^{\text{optimal}}$ = 542.63 Hz (0.017%). The change in rise distribution is illustrated in Figure 4 where the values are in meter and shall be added to the initial rise to get the distribution at optimum.

Discrete masses

In this case the wood material in the top was replaced with a reinforced plastic material composed by glass fiber and polycarbonate with the following properties: $E_1 = 3.97 \cdot 10^{10}$ Pa, $E_2 = 1.24 \cdot 10^{10}$ Pa, $G_{12} = 3.46 \cdot 10^9$ Pa, $\nu_{12} = 0.19$, and ρ = 2071 kg/m^3. The original thickness of the wooden top was decreased by 20% while the original rise was unaffected. At each node a positive point mass was taken as variable. Symmetry with respect to the vertical mid-axis was utilized. The initial eigenfrequencies was determined to: f_1 = 310.51 Hz (+9.6%), f_2 = 541.93 Hz (+6.1%), and f_3 = 636.58 Hz (+17.3%). The optimization process converged to the following eigenfrequencies: $f_1 = f_1^{\text{optimal, m}}$ = 271.79 Hz (-4.1%), $f_2 = f_2^{\text{optimal, m}}$ = 487.39 Hz (-4.6%), and $f_3 = f_3^{\text{optimal, m}}$ = 552.51 Hz (+1.8%). Following this optimization, with optimally placed discrete masses at specific points, an optimization with the shell thickness as variable (formulated as above) was performed which converged to the following eigenfrequencies: $f_1 = f_1^{\text{optimal, m+t}}$ = 280.76 Hz (-0.9%), $f_2 = f_2^{\text{optimal, m+t}}$ = 508.14 Hz (-0.5%), and $f_3 = f_3^{\text{optimal, m+t}}$ = 545.70 Hz (+0.6%).

DISCUSSION AND CONCLUSION

The optimization problem was to minimize the difference in eigenfrequencies for violin tops with different materials. Three eigenmodes were studied, according to Figure 2, and the problem was formulated as: minimize the quadratic sum of the difference of each eigenfrequency multiplied by a penalty parameter, see equation (1). For the case with variable shell thickness, the difference in eigenfrequencies are within 0.004% and for the case with the arching variable, the difference are within 0.049%. In both these cases the material in the top was spruce with minor difference in quality and the results indicate that it could be possible to compensate for this difference in material by changing the thickness and arching distribution. In the case with discrete masses as variables and a reinforced plastic material in the top the optimization process was not able to converge as close as the two earlier cases. With a following optimization with the shell thickness as variable the eigenfrequencies came to be within 0.9%. The boundary condition used in these analyses was not chosen to simulate a real situation for a violin top, here our primarily purpose was to investigate the possibilities in using optimization to compensate for difference in material properties. Although the results are inspiring much work remain to be done in order to understand the behavior of a good violin and to be able to, in some sense, guide the manufactures of violins.

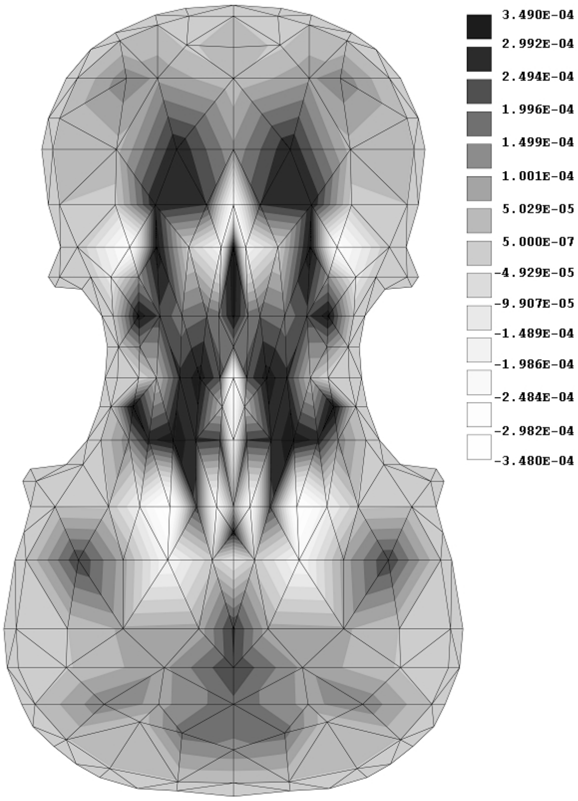

Fig. 3. Change in thickness distribution

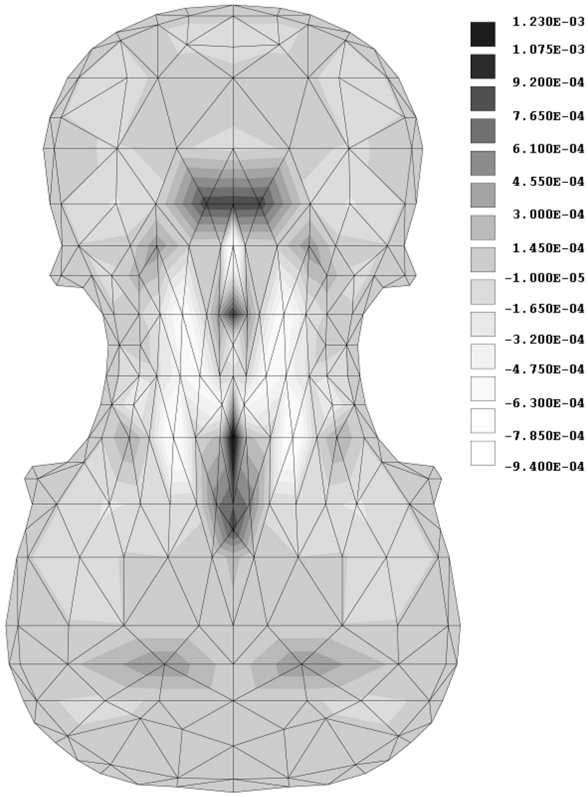

Fig. 4. Change in arching

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