ON ARCHING SHAPE AND VIOLIN TONE

Part 2
A quantitative appraisal of the hypothesis relating LSV, arching shape and radiated sound

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Abstract

Vibration measurements are presented, which were made to see what support there might be for the hypothesis offered in Part 1. The admittance of the bridge to the internal forces applied to it by the string is separately measured for longitudinal string vibration (LSV) and transverse string vibration (TSV). The input mechanical power to the violin from the TSV is estimated. The mechanical power exchanged at the bridge and saddle by longitudinal string vibrations LSV is shown to be comparable in magnitude with the input mechanical power. The effect on the radiated sound caused by changes in the arching shape parameters is measured at 196Hz intervals. This can be optimised by control of the arching shape parameters. The radiated sound above 2kHz is shown to relate more closely to the LSV than the TSV.

Experiments by driving the string and making objective measurements

In an attempt to see what objective evidence there is for the hypothesis presented in Part 1, a series of experiments were done. The hypothesis is that the input power to the violin comes from the bowing of the string. The string holds a reservoir of energy releasing small quantities to the body at each cycle of TSV, where it dissipates as mechanical loss or radiated sound. The transfer of energy to the body creates a complex vibrational response in the violin. This involves forces in the string (secondary LSV). The string must react and apply equal and opposite constraining forces to the body. Thus the modes in the violin are driven by power from TSV but determined by forces from both TSV and LSV acting together on the body. These forces will excite best those modes that are nearby in frequency and have large displacements in the direction of the forces. In this way LSV influences the modal behaviour of the violin.

For this to be so, it might be expected that the mechanical power exchanged between the string and the body through LSV would need to be large enough relative to the input TSV power to influence the mode shapes that are driven by TSV. This is not necessarily so since the effect of LSV force on mode shapes might be purely reactive with no power exchange. However, the first experiment was to measure the mechanical power input from the string to the body by TSV, and the power exchanged between the string and the body through LSV. The mechanical power exchanged between the string and the bridge can be derived from the admittance of the bridge to TSV and LSV. This requires that the TSV and LSV forces in the strings be measured, and the admittance of the bridge must separately be found for TSV and LSV. Since a measurable TSV force was required, the excitation needed to be via a string vibrating at resonance. This excitation had to be sustainable to enable a succession of measurements to be made (using a two channel analyser). Additional experiments were done to see if there was a difference in the sound radiation of violins of different EAR and different deviation. This required that the string excitation be repeatable to enable an identical excitation to be applied to each of the violins being compared.

A sustainable and repeatable excitation of the string was achieved by driving the open G string at its fundamental resonance frequency by a shaker. A shaker is an electrically driven vibrator. This was contrived to drive the string transversely (normal to the rib) through a wire probe that acted on the string at a point about 10 mm from the nut end of the string. The shaker imparted a single frequency displacement to the string. Of course a small reaction force was generated, but the shaker was acting essentially as a displacement source.

A number of preliminary experiments were done using the shaker to study the vibration set up in the string. If a string is tensioned between two fixed points and driven at resonance by a shaker with a single frequency input, it would vibrate predominantly at a single frequency, but only when the displacement is very low. As the displacement is increased a wide range of higher harmonics appears in the string transverse vibration. If the string is then put on a violin and driven the same way, the harmonic content of the string vibration, both transversely (TSV) and in tension (LSV), is much greater. The higher harmonics did not appear in the measured input displacement produced by the shaker. Regardless of the reason for the harmonics the fact remains that they were there. Thus the shaker was used to drive the string at its fundamental resonance only, but the resulting string vibration contained a range of harmonics sufficiently large to be measurable and usable as an excitation for all the experiments. Figure 1 shows the string displacement at each of the harmonics for both a typical bowed G string (assuming the displacement declines as 1/n) and for a shaker driven G string with a first harmonic transverse displacement of 3.2mm.

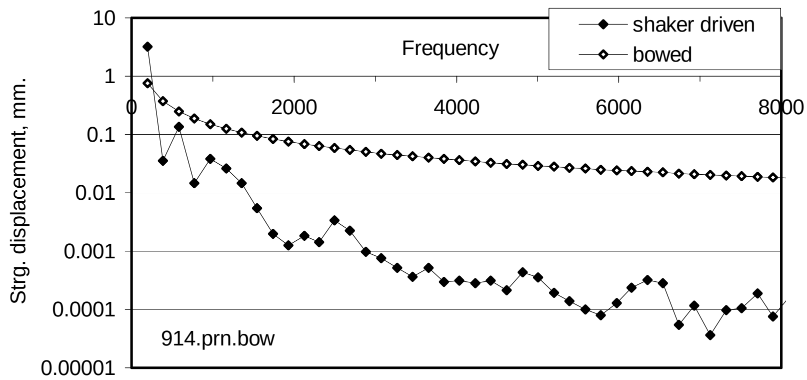


Figure 1. String displacement of a shaker driven open G string compared to that of the bowed string. The peak heights at the harmonics are shown as points (at 196Hz intervals), and the points are joined by lines to improve the clarity of presentation.

It will be seen that the shaker driven string displacement in the first harmonic is higher than that of the bowed string but in the harmonics above this they are well below that of the bowed string. This is not a problem if the system is linear. The relatively low contribution from the non-linear primary LSV input was not believed to affect significantly the linearity of the system. This was checked by measuring the power transferred per force squared, at the tailgut, for both a shaker driven violin and a bowed violin. They agreed quite well although they are dependent on the ratio of LSV/TSV, which is slightly non-linear. The excitation was, with a little practice, made to be both sustainable and repeatable. The sustainability was often prone to wander off resonance but if this happened the measurements were discarded and a fresh start made.

A transducer at the bridge notches would not separately measure the TSV force and the LSV force. So, in these experiments the transverse displacement of the string and the TSV force on the bridge were inferred from measurements of the string transverse velocity. The velocity was found by placing a strong ferrite magnet above the G string and another below it, close to the free end of the fingerboard 50mm from the bridge, so that the plane of string vibration cut across lines of magnetic force. This induced an EMF in the string, which was proportionate to the string velocity at the magnet gap. The string displacement amplitude in the first harmonic (6.4 mm peak to peak) was measured by a scale rule and the system calibrated. Corrections were made for the position of the magnet in the string length and a further correction was made for the effect due to the magnet being relatively wide in relation to the wavelengths of the higher harmonics.

A force transducer placed at the interface, where the tail gut pulls on the tailpiece, measured the total LSV force. The LSV force in the group of four strings was assumed to be the same at the bridge as was measured at the tailgut. This would be so if there were no resonances in the tailpiece or the strings between the tailgut and the bridge. The longitudinal resonance frequency, in the second harmonic, of the G string between the tailpiece and the bridge was calculated to be about 18kHz, and all other resonances would be higher. If the G string resonated over the length from the tailpiece to the nut its second harmonic frequency would be about 2650Hz, but the phase shift between the tailpiece and the bridge would only be about 1/7 of 180 deg. It was therefore believed that over the frequency range that mattered the LSV at the tailgut was little different to that at the bridge. The vertical LSV force on the bridge was inferred from the LSV force in the string.

The accelerations at the bridge feet and the saddle were measured with very small and lightweight accelerometers that were placed on the belly immediately in front of each bridge foot on the fingerboard side of the bridge.

The Admittance of the bridge to TSV and LSV forces applied by a vibrating string

Admittance graphs are often shown for an externally applied force to the bridge. These are thought to simulate the effect of a TSV force on the bridge. The admittance is the velocity resulting from unit-applied force (admittance is the term used by physicists, but means the same as the acoustical term, mobility). It is a complex quantity that can only be represented by two graphs, magnitude and phase or real and imaginary parts. The admittance of the bridge (with the strings on) to the TSV force and the LSV force were separately measured, the forces having been applied as internal forces from the string to the bridge rather than an external force.

The vertical accelerations at the bridge feet were used to infer the vertical and transverse velocity of the top of the bridge. This assumed that the bridge did not deform in its own plane. Cremer [1] concludes from the results of Reinicke [2] that this is a valid assumption up to 2kHz. Above this in the “bridge hill band” the effects of the natural resonance of the bridge may intervene. Comparison of Reinicke’s measurement of the bridge impedance on a rigid base with measured admittances of

the bridge on the violin body show that the admittance is controlled by the rotational stiffness of the bridge in its own plane at admittance minima (anti-resonances of the body) and by the body at other frequencies. Consequently, within the most important regions of the admittance curve for excitation by a string the bridge may be considered to be rigid in its own plane (up to about 3kHz). Durup and Jansson [3] have shown that in this band "mainly the vibrations of the bridge feet set the measured mobility". They also show that "a plate bridge with no cutouts can replace a normal bridge with little effect on the mobility" in this band. It is conceded that inferring the velocities of the bridge top from the foot accelerations may nevertheless introduce some uncertainty in the results above 2kHz. It also assumed that there was no transverse movement at the bridge feet. The top velocity was split into two components: a vertical bouncing motion and a horizontal translation. Bridge top vertical motion does not result purely from the LSV force, or the bridge top translation purely from the TSV force, so these admittances are not completely independent. For this reason they are referred to as pseudo-admittances. This is because an external force excites the violin in a manner that is significantly different from that of the vibrating string . Boutillon and Weinreich's results [4] imply that those string forces on a bridge that have components in directions other than in the plane of the bridge, can be responsible for power transfer between the strings and bridge other than that conventionally assumed. Data was only available at 196Hz intervals but have been joined up for clarity of presentation only.
This is shown in figure 2.

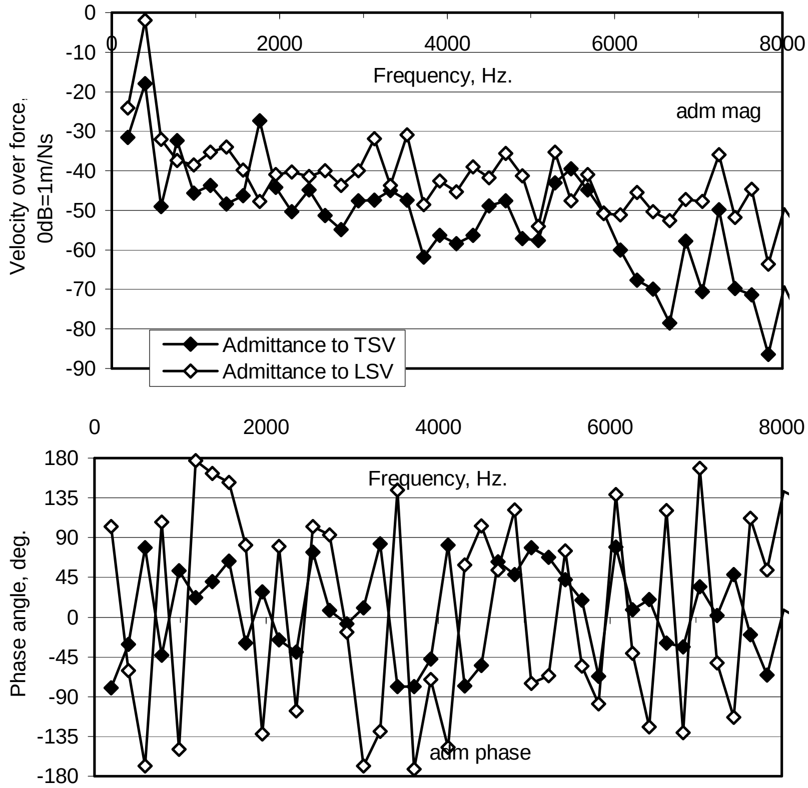


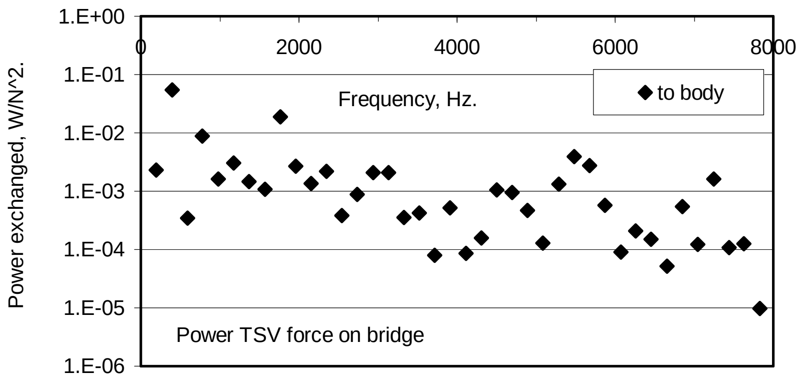
Figure 2. The magnitude and phase of the pseudo-admittance of the bridge to the forces applied by a transversely vibrating string. The data was only found at 196Hz intervals and are shown as points. The points are joined by lines to improve the clarity of presentation.

Graphs showing the admittance to an <u>external</u> force at the top of the bridge do not show the same magnitude as figure 2 except near the first harmonic where there is agreement. The admittance shown in figure 2 is for a violin excited by a vibrating string. The string would apply TSV forces at the bridge and at the nut (for an open

string). The nut is widely assumed to have a low admittance compared to the bridge but this may not be so. The modes of the violin often involve considerable nut movements and could offer significant admittance to a force applied at the nut. The violin is a system and its modes are independent of the form of excitation. But a different form of excitation will apply different modal forces and so the system modes will be excited to different relative amplitudes and phases. Excitation via the string will favour those modes that involve large string amplitudes at the position and in the direction of the exciting force. Excitation via an external force on the bridge will excite best those modes in which the bridge top moves strongly in the direction of the applied force. So the response cannot be expected to be the same.
Of course another possible explanation of the difference is that there is some mistake in figure 2. We can check this by referring to figure 4. This shows the power input to the bridge by TSV, which was derived from the admittance. The radiated sound power is also shown (the method of its measurement is explained later). The measurement of sound power was not calibrated but is shown in figure 4, scaled to give a best fit to the input TSV curve at the low frequencies. As the frequency rises the radiated sound power curve rises above the input TSV power. Thus the fraction of vibrational energy radiated F_{rad} (the radiated sound power over the input mechanical power from the vibrating string) increases with rising frequency. This has been shown by Bissinger [5] to be generally so for violins, and he shows F_{rad} increases from 0.2 to 0.7 (approximately) as the frequency rises from 400Hz to 4000 Hz. The measurement of the radiated sound power is in every way independent of the measurement of the admittance. That there is general agreement between the shape of the graphs for input TSV power and the output sound power (allowing for an increase in F_{rad} with rising frequency) shows that the admittance curves in figure 2 (from which this is derived) cannot be significantly incorrect in their shape.

The power exchanged between the string and the body

Admittance graphs are difficult to interpret, but from them the power exchanged at the input ports can be found. The time-average power exchanged at the input ports is the time-average product of force and velocity, or (for harmonic vibration) the real part of the admittance times half the square of the magnitude of the input force. When vibrational energy is transferred from one system to another the rate of transfer of energy may be quantified in terms of mechanical power. This is shown in figure 3.



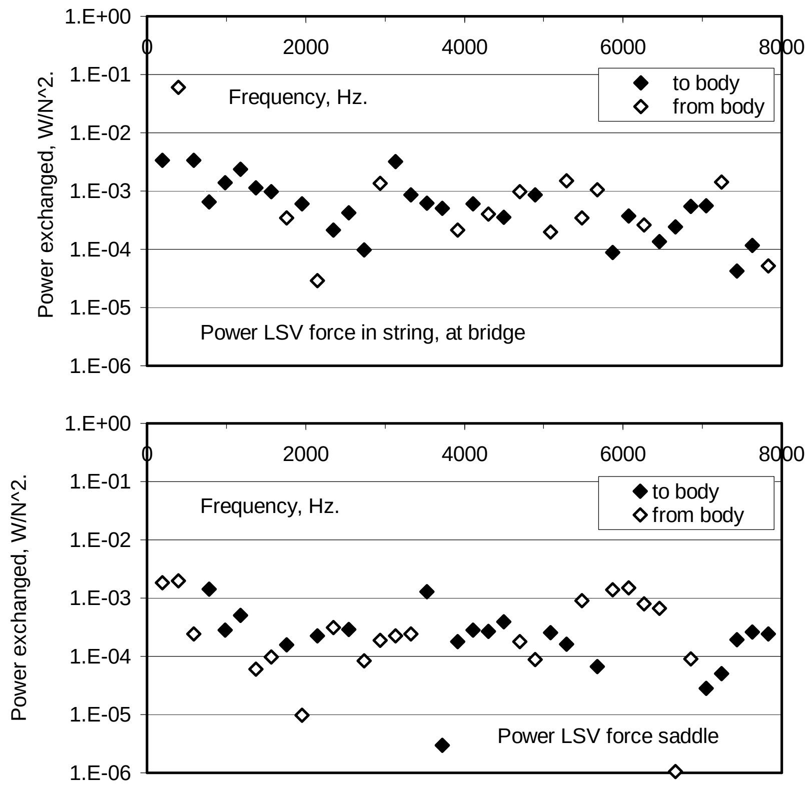


Figure 3. The power in Watts exchanged per unit force squared. Shown for a TSV force and an LSV force at the bridge, and for an LSV force at the saddle. The data is presented at 196 Hz intervals only.

The data points in these graphs are not pseudo, but the actual power exchanged per unit input force squared. No measurement was made of the power exchanged between the string and the body at the nut or stopping finger. The power resulting from TSV is always positive in that it puts power into the body from the string. The energy flow from LSV at the bridge, or from LSV at the saddle can be into or out of the body. For example, a rocking bridge might lift the group of strings taking power out of the body into the string at the bridge, and the resulting increase in string tension may put power back into the body at the saddle and/or the nut.

If LSV plays a significant role in controlling the vibrational behaviour of the body, it is reasonable to assume that it would influence the distribution of vibrational energy flow. The data shown in figure 3 was applied to a bowed open G string (assuming linearity) to produce figure 4. This shows the power input to the bridge by TSV, compared with the power exchanged between the string and the bridge by LSV and the power exchanged between the strings and the saddle by LSV. The radiated sound power is also shown.

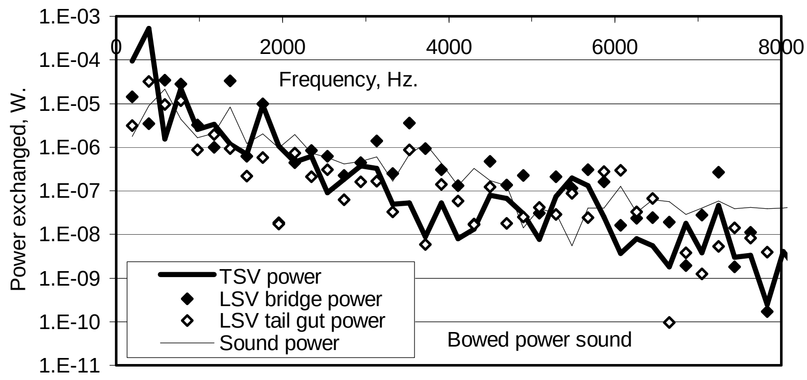


Figure 4. Power input to the body by TSV at the bridge, compared with the power exchanged at the bridge and saddle by LSV (the TSV data points and the LSV direction of power transfer are omitted for clarity). The radiated sound power (uncalibrated, and arbitrarily positioned) is shown for comparison (data points omitted for clarity).

It is the magnitude and direction of the internal forces within the violin body, which determine which modes that are excited. The energy associated with the TSV force and LSV forces within the body are comparable. Generally the proportion of power redistributed by LSV increases with rising frequency.

The effect of arching shape on the radiated sound

Using wood cut from the same board we made three violins, identical in all respects (all with the value of EAR we normally use), except that one had a low deviation (about 9% of the EAR), one had a medium deviation (about 14% of the EAR) and the third one a high deviation (about 19% of the EAR). See Part 1 for an explanation of the terms EAR and deviation. By interchanging the bellies on two of these instruments we were able to make three violins of differing EAR, one with the EAR we normally use (referred to as medium EAR), one with an EAR 5% lower, and one with an EAR 5% higher. It should be noted that a 5% variation is only a very small amount and shows the sensitivity of tone to the EAR. These violins were in all other respects as identical as we could make them and had closely matching plate thicknesses and tunings.

The effect of EAR The violin with the normal EAR was much easier to play, was completely open from new and was greatly preferred by both listeners and players. The other two exhibited the high and low EAR characteristics listed above, were harder to play and not preferred by listeners. Figure 5 shows the sound radiated by the three violins (driven by a shaker), the peak heights have been joined up and the data points at 196Hz intervals have been omitted for clarity. The sound pressure was measured in a large, moderately reverberant room of which the reverberation time varied little with frequency. Therefore the mean square pressure was proportional to the sound power. The microphone was suspended from its cord at an average distance of 7 feet and set to swing through an arc about 7 feet across during the gathering of the samples of sound. This simple device enabled a fair spatial and temporal average of the squared radiated sound pressure to be picked up. An identical set-up was used for all the violins tested to give a valid comparison.

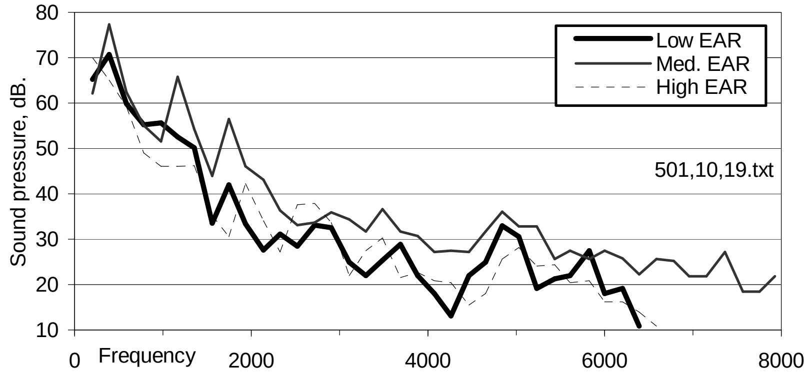


Figure 5. Sound pressure level radiated (uncalibrated), equal first harmonic string displacement. Shaker driven G string. Shown for violins of differing EAR. Data points (at 196Hz intervals) omitted for clarity

Over nearly all the range the medium EAR violin radiated more sound per unit first harmonic string displacement. Importantly however, at the first harmonic frequency the medium EAR violin radiated slightly less sound than the high and low EAR violins. Above about 6,500 Hz, the low and high EAR violins did not radiate enough sound to be distinguishable from background noise.

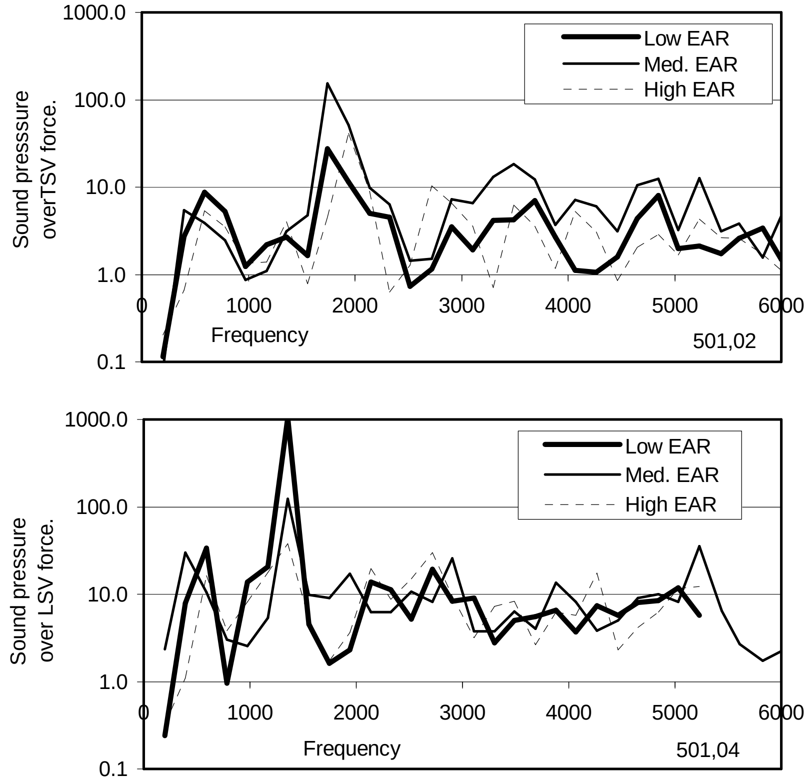


Figure 6. The radiated sound pressure divided by the TSV force (above) and by the LSV force (below). Data points (at 196Hz intervals) are omitted for clarity.

Figure 6 shows the radiated sound pressure divided by the LSV force and by the TSV force. Above about 2000Hz the modal overlap in a violin increases. The applied forces will excite those modes that have displacements in the direction of the force. Figure 6 shows that the radiated sound above 2000Hz relates much more closely to the LSV force (radiated sound does not vary with the EAR) than the TSV force (radiated sound does vary with the EAR). The inference is that LSV has a major influence on what modes the violin radiates from and this influence increases with rising frequency.

The effect of deviation The deviation determines the force on the end bouts cross arches. Logically, if the deviation is doubled the force on the arches might be doubled and the radiated sound might double. Because the ear is a non-linear listening device, a doubling of the sound pressure is such a small difference that it is not readily perceptible unless the change is presented to the listener in quick succession, but it may be measurable. To investigate this more objectively the three violins of differing deviation (but the same EAR) were tested by driving the open G string to the same first harmonic displacement, with a shaker, and measuring the radiated sound.

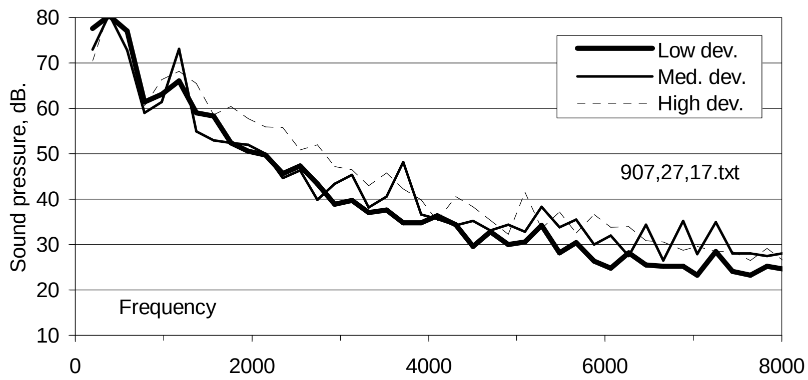


Figure 7. Sound pressure radiated (uncalibrated), equal first harmonic string displacement. Shaker driven G string. Shown for violins of differing deviation. Data points (at 196Hz intervals) omitted for clarity.

Figure 7 shows that the radiated sound of the high deviation violin is nearly always more than that of the low deviation violin with the medium deviation violin mainly being in between. But importantly again, at the first harmonic the order is different, the radiated sound is greatest when the belly end bouts cross arch is the lowest (the low deviation violin). The radiated sound divided by the TSV force had a tendency to vary with the deviation, but the sound divided by the LSV force did not. This again showed that the LSV is strongly influencing the radiated sound.

The violins were placed in a broadband sound field and the LSV force at the tailgut was measured. This showed that LSV force could be excited reciprocally, indicating that it is strongly coupled to the most efficiently radiating modes. Comparing the violins of differing EAR showed that the violin of medium EAR induced a greater LSV force than the other violins between 2kHz and 8kHz. This suggests that above 2kHz it was most closely coupled to the most efficiently radiating modes, and that the effectiveness of the coupling depends on the EAR. The violins of differing deviation (and the same EAR) all induced the same LSV indicating they were equally well coupled to the most efficiently radiating modes [6].

Conclusion

The approach taken in the research reported in these two papers, was to excite the violin by vibrating the string at resonance and then taking a look inside the whole system (the violin with the strings on) to try and observe what is happening at the interface between the strings and the body. Estimates have been made of the mechanical power exchanged across these interfaces.

The input power to the system comes from an external force applied to the string (bow or shaker). The string holds a reservoir of vibrational energy releasing small quantities to the string supports (bridge and nut) at each cycle of TSV. The time-average TSV power flows from the string to the body (within the system) where it dissipates as mechanical loss or radiated sound. The input TSV power to the body through the bridge has been estimated. The input TSV power through the nut has not. The primary LSV power from the string to the body was shown to be small.

The transfer of energy to the body creates a complex vibrational response in the system (body plus string). The response involves a longitudinal vibration in the four strings (secondary LSV). This has been measured and the power exchanged between the string and the body by LSV has been shown to be comparable in magnitude with the incoming TSV power through the bridge. The longitudinal forces involved in the strings will react and apply equal and opposite constraining forces to the body. Thus the modes in the whole system are driven by power from TSV but determined by forces from both TSV and LSV acting together on the body. These forces will excite best those modes (of the whole system) that are nearby in frequency and have large displacements in the direction of the forces. In this way LSV influences the modal behaviour of the violin. It was demonstrated experimentally that;

1. LSV can be excited reciprocally by excitation of the violin by a sound field and that the strength of the link between LSV and the sound field was dependent on the EAR (arching shape) above 2kHz.
2. The radiated sound depends on the EAR and in a different way on the deviation (arching shape). The qualitative analysis in Part1 indicated that the body response to LSV would depend on the EAR.
3. The sound radiated above 2kHz relates more consistently to the LSV than the TSV.

These observations together support the conclusion that LSV forces within the body influence the radiated sound particularly above 2kHz, and that this influence has a sensitive dependence on the EAR and to a lesser extent on the deviation.

The experimental evidence presented does lend considerable support to the hypothesis proposed in Part 1 of this paper. These papers present a preliminary view of a plausible explanation of some of the relationship between arching shape and violin sound and an overall concept of how the violin uses its characteristic shape to enhance the radiated sound. If the violin could do without LSV and just run on TSV, Felix Savart's trapezoidal violin may have caught on. The arching of the plates and the hourglass outline shape are all essential components of the mechanism that enables LSV to move the plates and radiate sound.

References

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